京都大学
KYOTO UNIVERSITY

2024 年度 SP+ Fund 報告書（General）
Project Report: SP+ Fund 2024 (General Program)

プロジェクトの基本情報／Key Project Information	
課題名（英語） Project name (in English)	Efficient machine learning algorithm for quantum systems with anyonic excitations
期間／Period	From 2024/07/01 Until 2025/03/15
主な研究分野 Main research fields	Physics, Informatics
活動内容 （該当するものに全て✓してください。） Activities to be funded (check ✓ all applicable items)	☐ 研究ワークショップ、会議、ラウンドテーブル、シンポジウム等の実施／Research workshops, conferences, roundtables, symposiums, etc. ☑ 共同研究や研究打合せにかかる渡航・招へい Travel/invitations for collaborative research or research meetings ☐ その他（具体的に）／Other (please specify) ( )
区分／Type of collaboration	☑ Bilateral ※本学と SP 校との２機関で実施するプロジェクト (Project conducted by Kyoto University and one SP institution) ☐ Multilateral ※本学と SP 校に加え、さらに１機関以上 (Project conducted by Kyoto University, an SP institution, and one or more additional institutions)
実施場所／Location of implementation	☑ 京都大学／Kyoto University ☑ その他／Other location (Hamburg University)

申請者（京都大学）／Applicant (Kyoto University)	
姓／Family name	PETERS
名／Given name	Robert
職名／Position	Associate Professor
所属部局 Faculty/dept. of affiliation	Department of Science

SP 校のプロジェクト代表者／Representative from SP institution	
姓／Family name	POSSKE
名／Given name	Thore
職名／Position	Head of research group (permanent)
所属大学／Institution	☐ ボルドー大学／University of Bordeaux ☐ ウィーン大学／University of Vienna ☐ チューリヒ大学／University of Zurich ☑ ハンブルク大学／University of Hamburg ☐ 国立台湾大学／National Taiwan University
所属部局	I. Institute for Theoretical Physics & The Hamburg Centre for Ultrafast Imaging

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SP 校のプロジェクト代表者／Representative from SP institution	
Faculty/dept. of affiliation	

その他のプロジェクト代表者（Multi の場合）／Representative from other collaborating institution (in the case of multilateral projects) ※	
姓／Family name	
名／Given name	
職名／Position	
所属大学／ Institution	☐ ボルドー大学／University of Bordeaux ☐ ウィーン大学／University of Vienna ☐ チューリヒ大学／University of Zurich ☐ ハンブルク大学／University of Hamburg ☐ 国立台湾大学／National Taiwan University ☐ その他／Other （機関名／name of institution: ）
所属部局 Faculty/dept. of affiliation	

※4 機関以上によるプロジェクトの場合は、必要に応じて欄を追加願います。
If the project involves four or more institutions, please insert additional fields as required.

プロジェクトの実施内容／Summary of the project 公開されている関連リンクや、フライヤー、プログラム、報告書、広報記事等の提出をもってして代えることも可能です。 This could be substituted by submitting publicly available relevant links, flyers, programs, reports, publicity articles, etc.
Using machine learning and especially neural networks, we study excitations in topological quantum spin systems that cannot be analyzed with conventional numerical techniques. Numerically analyzing quantum spin systems poses a massive problem to conventional methods, such as exact diagonalization or tensor networks, because of the exponential growth of the Hilbert space in quantum systems, thus limiting conventional numerical techniques to relatively small sizes in two (or three) dimensions. On the other hand, neural networks have demonstrated a remarkable ability to find and describe strongly entangled quantum states in large quantum systems, because of their ability to generalize and interpolate data. We have used neural networks as trial wave functions with variational quantum Monte Carlo to find the ground state of the Toric Code, a famous toy model hosting a topologically nontrivial ground state that can be used for topological quantum computation and error correction. In our project, we have confirmed the known results for the pure Toric code, demonstrating the ability of neural networks to find and describe the exact topological ground state. We also confirmed the ability of the neural network to correctly describe braiding and the action of loop operators that are necessary for quantum computation. We then extended the Toric code, including perturbations such as a magnetic field or the Heisenberg interaction. This is an important step as, in experimental situations, an exact reproduction of the Toric code will not be possible but will include small perturbations. Using neural networks, we were able to analyze the effect of such perturbations on the topological ground state. We found that for intermediate strong perturbations, the topological ground state persists, and topological quantum computation remains possible. However, we found that the loop operators that are used to manipulate the Qubits must be changed when the Heisenberg interaction is present. This is a significant result as it shows that perturbations make modifications of the operators necessary, which will be used as gates in topological quantum computation. These research results were obtained as a collaboration between Kyoto University and Hamburg University. Here, it was invaluable for the researchers from Kyoto to visit Hamburg, as the face-to-face discussions facilitated a deeper exchange of ideas and insights regarding these results. Besides discussions concerning the obtained results, we also discussed the future prospects of this project.

今後の展望／Prospects for future research collaboration
During the visit of researchers from Kyoto University to Hamburg University, we also discussed future research directions in this project. Particularly interesting in this respect is the direct simulation of error correction in topologically nontrivial systems such as the Toric code. While error correction is theoretically established in the exact Toric Code, it is unclear whether it is possible under more realistic conditions, including real-time evolution, perturbations of the system, and erroneous gates. Using neural networks and time-dependent variational Monte Carlo, we can analyze error correction directly under more realistic conditions. Furthermore, while the Toric Code is an effective model, more realistic models with topologically nontrivial ground states for topological quantum computing exist. Thus, we also want to extend our research to analyze these models, such as the Kitaev hexagonal model or quantum spin liquids on the Kagome lattice. The stay of the Kyoto researcher also enabled discussions about the simulation of quantum thermodynamics with neural networks. There, the calculation of the entropy or the partition function plays a central role that poses problems which have not yet been overcome. In the future, we would like to employ specific mathematical techniques to bring forward the research in this area.